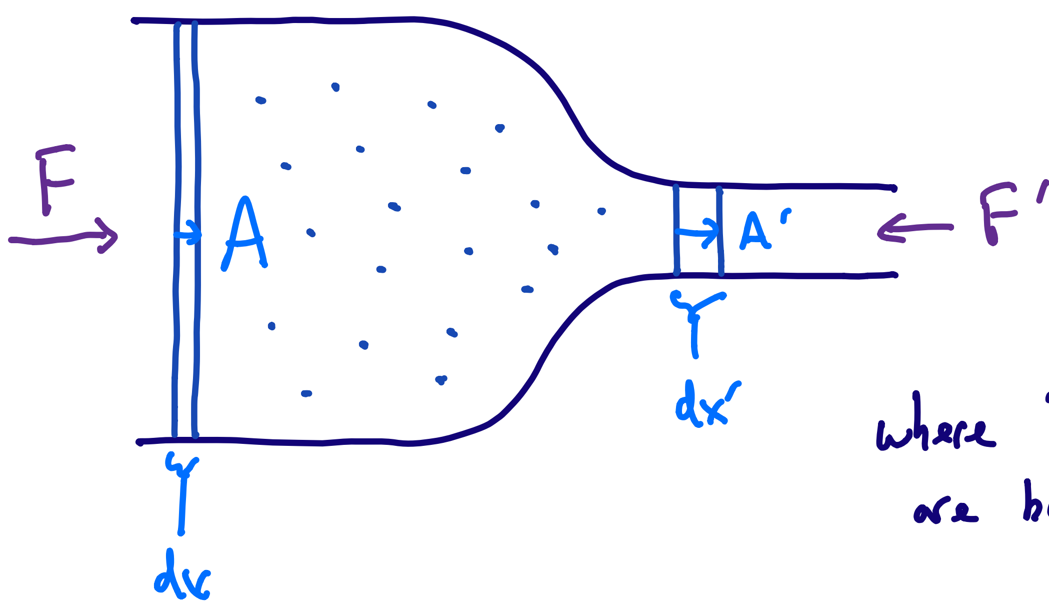
Lecture 17: Fluid Mechanics

Incompressible Fluid



where F and F' are balanced

Each piston moves a distance dx and dx', so the corresponding displaced volumes are

$$dV = dx\,A = dx'A' = dV'$$

the definition of "incompressible"

Dividing by dt we obtain $\frac{dx}{dt}A = \frac{dx'}{dt}A'$

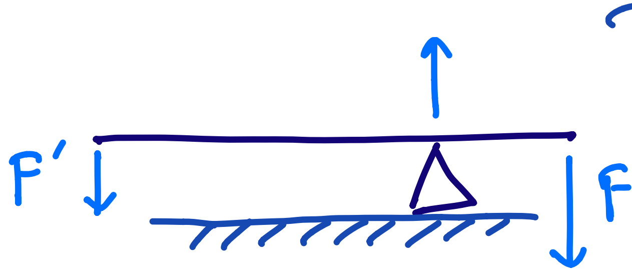
So decreasing area will increase velocity. This is how whistling works.

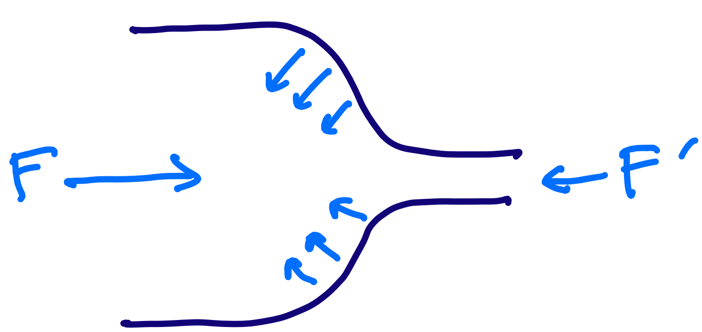
⟪demo: "three floating balls"⟫

Finally, note that force per area is fixed:

$$\frac{F}{A} = \frac{F'}{A'} = P \quad \left(\begin{array}{c} \text{pressure} \\ \text{"Pascal"} = \text{Pa} = \text{N}/\text{m}^2 \end{array} \right)$$

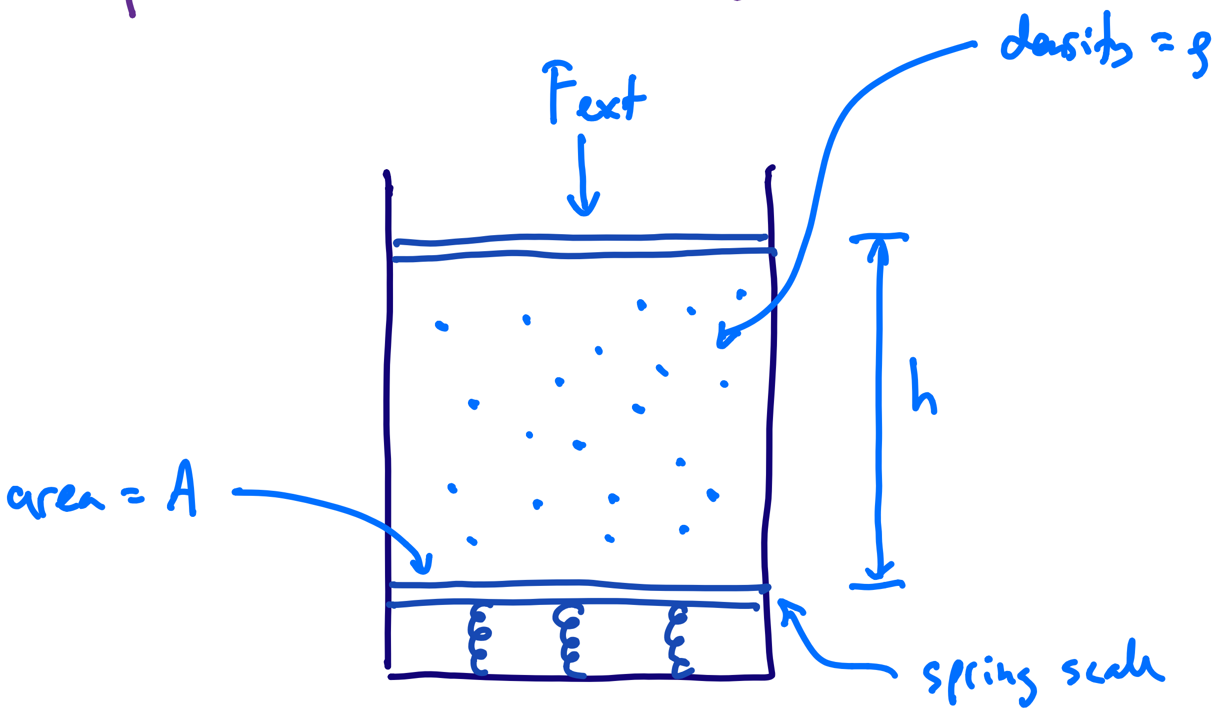
possible confusion: how is the system balanced if $F \neq F'$???





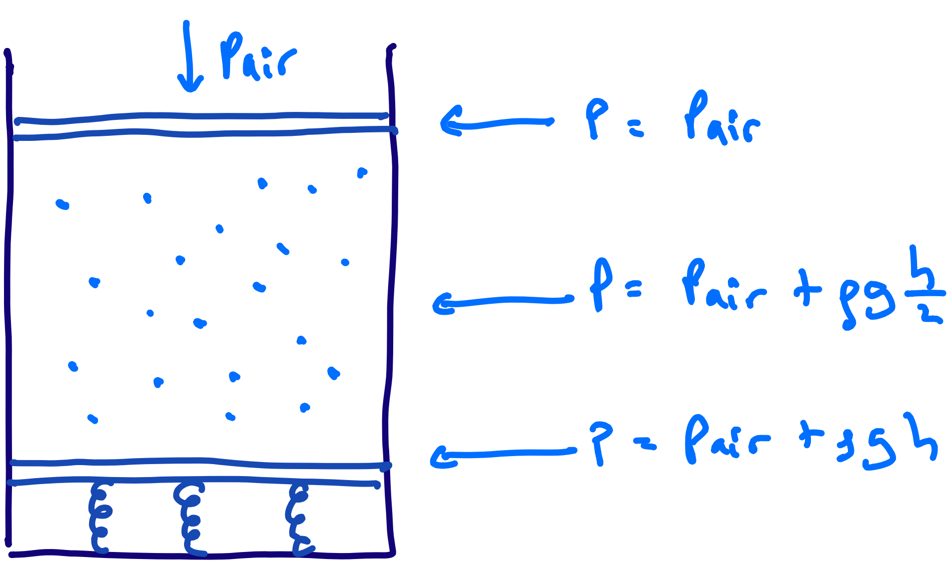
like a see saw there are other forces at play

Incompressible Fluid with Gravity



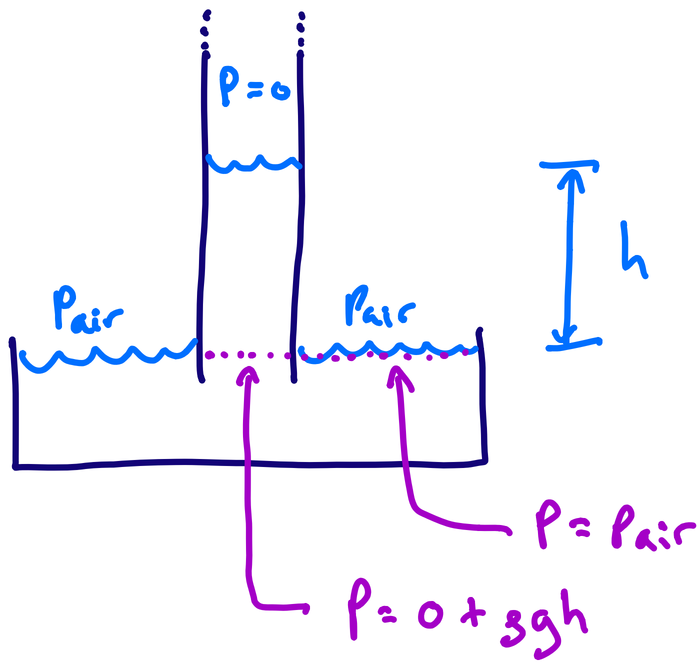
$$F_{spring} = F_{ext} + \underbrace{(\rho A h)}_{\text{mass of column of fluid}} g$$

$$P_{spring} = \frac{F_{spring}}{A} = P_{ext} + \rho g h$$



demo: "holey water bottle"

These pressure relations are exploited by a barometer to measure air pressure.



$$\Rightarrow \quad P_{air} = \rho g h \quad \text{so} \quad \boxed{h = \frac{P_{air}}{\rho g}}$$

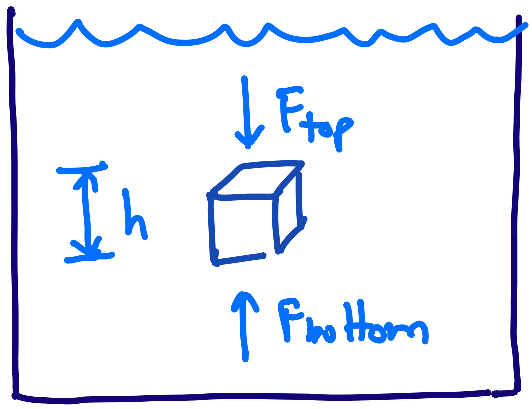
height measures pressure!

For $P_{air} = 1\,\text{atm} \equiv 101325\,\text{Pa}$ and $\rho_{water} = 1\,\text{g}/\text{cm}^3$,

$$\Rightarrow \quad h_{water} = \frac{101325\ \text{N}/\text{m}^2}{10^3\ \text{kg}/\text{m}^3\ 9.8\ \text{m}/\text{s}^2} = \underbrace{10.3\,\text{m}}_{\text{absolute height you can suck a straw!}}$$

Buoyancy

Consider a box in fluid.



$$F_{top} = P\ell^2 \quad \text{and} \quad F_{bottom} = (P + \rho_{fluid}\, g h)\,\ell^2$$

$$\boxed{F_{buoy} = F_{bottom} - F_{top} = \rho_{fluid}\, g V}$$

submerged volume of object

For an object to float requires:

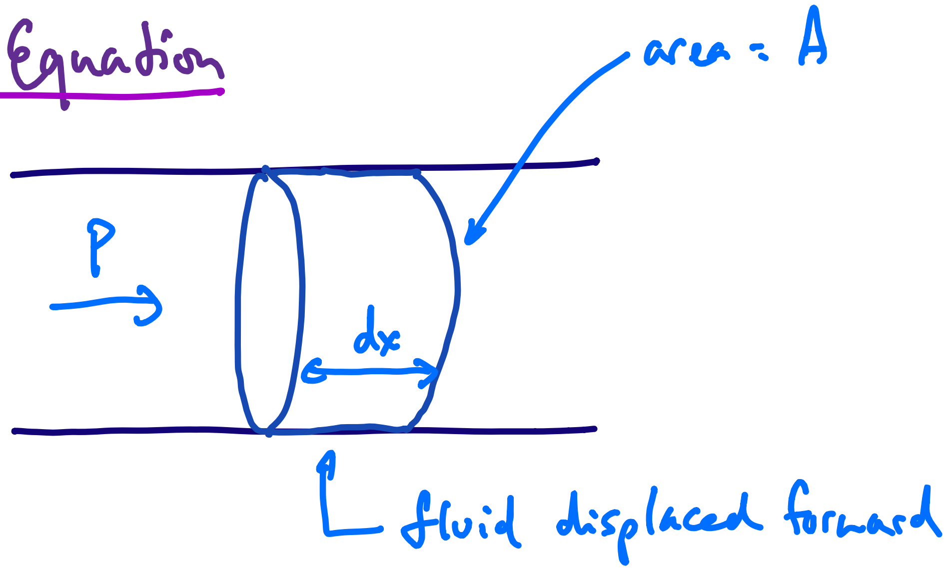
$$F_{buoy} > F_{grav} \rightarrow \rho_{fluid}\, \cancel{g} \cancel{V} > \rho_{obj}\, \cancel{g} \cancel{V}$$

$$\boxed{\rho_{fluid} > \rho_{obj}}$$

density of object must be less than fluid

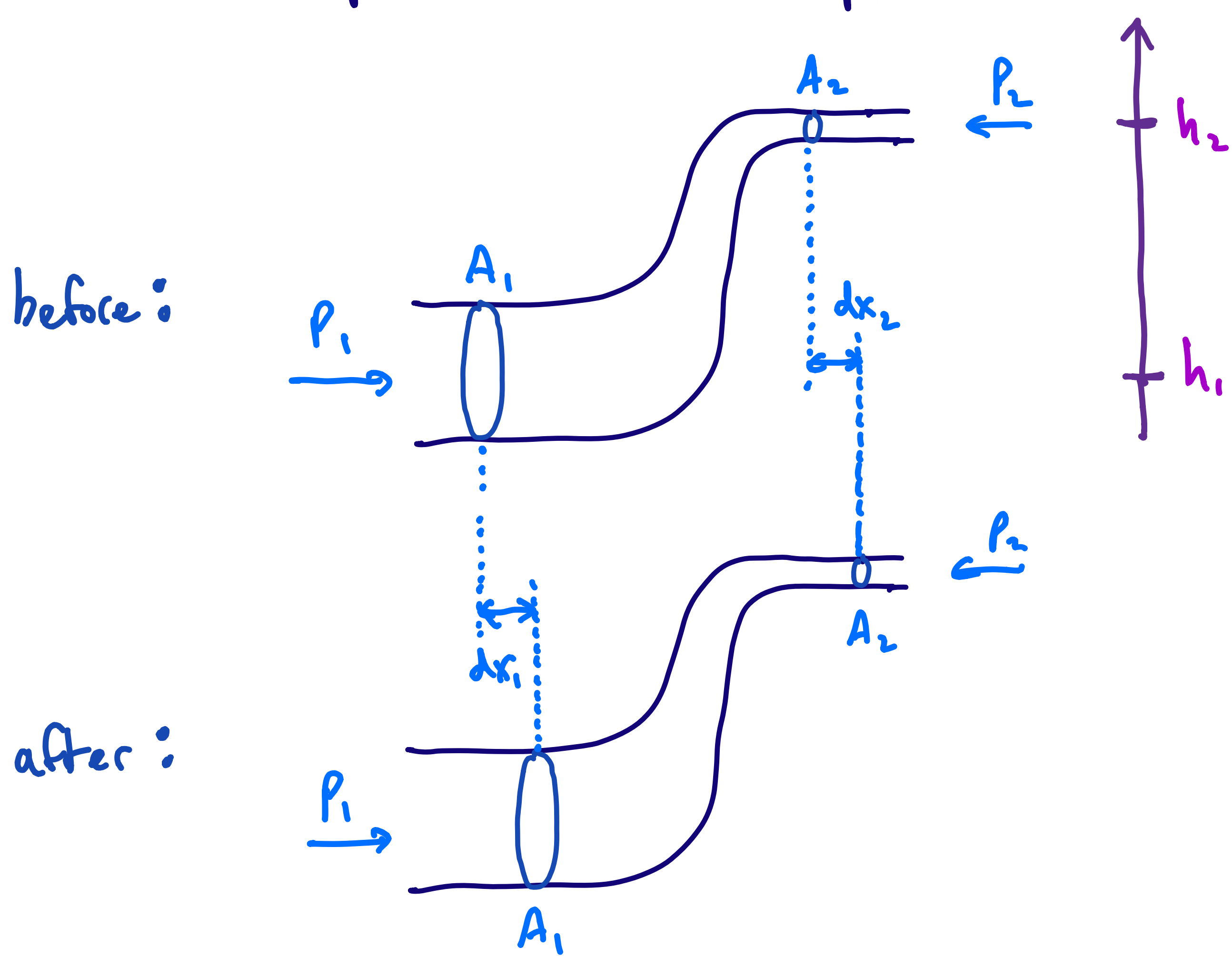
demo: "lead weight in fluid"

Bernoulli Equation



$$\text{work} = dW = F\,dx = PA\,dx = \underbrace{P\,dV}_{\text{extruded volume}}$$

Now consider displacement of fluid in a pipe.



$$dW = P_1 A_1 dx_1 - P_2 A_2 dx_2$$

$$A_1 dx_1 = A_2 dx_2 = dV$$

(incompressible)

$$= (P_1 - P_2)\, dV$$

Meanwhile, the change in energy is

$$dE = dm\, g\, (h_2 - h_1) + \tfrac{1}{2} dm\, v_2^2 - \tfrac{1}{2} dm\, v_1^2$$

$$= \rho\, dV$$

Equating work and energy we find

$$(P_1 - P_2)\, d\cancel{V} = \rho\, d\cancel{V} \left[g h_2 - g h_1 + \tfrac{1}{2} v_2^2 - \tfrac{1}{2} v_1^2 \right]$$

$$P_1 + \rho v_1^2/2 + \rho g h_1 = P_2 + \rho v_2^2/2 + \rho g h_2$$

$$\Rightarrow \boxed{P + \rho v^2/2 + \rho g h = \text{const}}$$

Bernoulli's Equation

increased velocity

$\updownarrow$

decreased pressure

or

decreased potential energy

⟪ demo: "floating balls" ⟫